1^{st} GIMO 2021

Gaussian Curvature

Day II Problems

Problem 4. We call a positive integer m *epic* if $\varphi(m)$ is not a power of 2. We connect two epic positive integers with an arrow, if and only if $\gcd(\varphi(a), \varphi(b))$ is a power of 2.

With justification, examine if it is possible to connect every two epic positive integers with a finite number of arrows.

Note 1. A power of 2 is a number of the form 2^k, where $k \geq 1$ is a positive integer.

Note 2. $\varphi(n)$ counts the number of positive integers less than or equal to n that are relatively prime with n.

Problem 5. A set of positive integers $\mathcal{A}$ is called *special* if there is a function f mapping the positive integers to the elements of set $\mathcal{A}$ such that

$$\frac{xf(x)}{yf(y)} \neq k,$$

for all integers $k \in \{2, 3, \ldots, 2021\}$, and all positive integers x, y.

Find the smallest positive integer n such that the set $\mathcal{S} = \{1, 2021, 2021^2, \ldots, 2021^n\}$ is special, or prove that such a positive integer does not exist.

Problem 6. Let ABC be a triangle with circumcircle ω and let the internal bisectors of angles A, B and C intersect ω at points M_A, M_B and M_C, respectively.

Let T_A be the midpoint of the segment M_BM_C, and let P_A be the point on line AT_A, which is closer to T_A, such that $AP_A = 4AT_A$. Let ω_A be the circle through A and P_A and tangent to AM_A. Define T_B, T_C, P_B, P_C, ω_B and ω_C similarly.

Show that the circles ω_A, ω_B and ω_C share one point in common.

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Submission Process I. You can submit your solutions on AoPS PM to Aritra12 and Orestis_Lignos if you have an account at AoPS. This is the most preferred way of submission and it is also beneficial to participants because on AoPS PM you are allowed to send solutions one by one in that single PM however you are not allowed so for the other two process. But obviously you can send day 1 and day 2 separately.

Submission Process II. For submitting Day II Solutions please upload solutions in the following link: `https://forms.gle/ZbNy8PPU8h54yUCR9`

Submission Process III. If you are unable to do any of the things above then just simply mail your solutions pdf to us on our mail gaussiancurv180@gmail.com